Location of Plastic Hinges in Columns of Steel Frames

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ABSTRACT: The New Zealand Steel Structures Standard has a seismic provision aiming to ensure that plastic hinges occur at the ends of steel columns rather than along their length. This provision often governs the column sizes in steel frames and concerns have been raised about the need for this requirement and the accuracy of the equation in the provision. This paper describes research carried out to evaluate this equation. Methods were developed from first principles considering second order and residual stress effects. Time history analysis was carried out with RUAUMOKO to examine the likelihood of plastic hinges forming away from the column ends. A computer program was also used to examine the applicability of this equation on different types of steel frame. A new equation is proposed based on the analytical results. It is found that the provision is not applicable to columns in the majority of moment-resisting frames. Columns in eccentrically braced frames, designed in accordance with the capacity design philosophy, often did not yield even at the column base during earthquake excitations. A simple check is proposed to determine whether yielding would occur under a design drift limit which can be used in conjunction with the proposed equation.

1 INTRODUCTION

The New Zealand steel structures standard, NZS3404, (Standards New Zealand 1997) has a seismic provision aiming to ensure that flexural yielding in columns only occurs at the ends. Yielding at the column ends, rather than away from the ends, is considered to be desirable because:

- column ends can be effectively braced to prevent local and lateral buckling,
- the rotational capacity is more reliable and occurs in the same place as in experimental tests on columns subject to flexural deformations, and
- the correct collapse mechanism and hinge rotational demands can be predicted more easily.

Clause 12.8.3.1(b) in NZS3404 specifies that for columns with various slenderness, λ, and end moment ratios, β_m, the design axial force, N^*, must satisfy Equation 1, where ϕ is the resistance or capacity reduction factor, and N_S is the nominal section capacity. Here, β_m is specified as zero for columns forming part of a seismic-resisting system and 0.5 for columns forming part of an associated structural system. This equation was originally developed by Lay (Lay 1964) to encourage yielding at the ends of members containing plastic hinges in frames designed using plastic analysis, as well as to ensure adequate rotational capacity of columns. It is applied to moment resisting frames, MRFs, as well as eccentrically and concentrically braced frames, EBFs and CBFs.

$$N^* \leq \phi N_S \left[\frac{1+\beta_m-\lambda}{1+\beta_m+\lambda} \right] \quad (1)$$

There is a need to re-evaluate Lay's equation and its applicability to columns in different types of steel frame for the following reasons:

1) The background to this equation is not clearly described.
2) The New Zealand steel code has a specific provision to ensure adequate rotational capacity of columns by defining limits on the maximum allowable axial force. Therefore, Lay's equation, covering both member end yielding and rotational capacity, may be more conservative than requirements that only consider yielding at the member ends.
3) The joint restraints and the load carrying mechanisms differ significantly between EBFs and MRFs.
4) The equation should only be applied to columns that are expected to yield. Columns which violate this provision but which are not expected to yield should not be increased in size to prevent yielding away from the ends.

For these reasons, a research project has been carried out at the University of Canterbury with the objective of re-evaluating Lay's equation and the assumptions and specifications adopted in this provision. An analytical method is developed from first principles to evaluate Lay's equation. A computer program 'Dr. Frame' (Dr. Software 1998) is used to assess the applicability of this equation to different types of steel frame. A time history analysis program 'RUAUMOKO' (Carr 2004) is used to examine the likelihood of plastic hinges forming away from the column ends in EBFs. A simple method is proposed to relate the axial force limit to inter-story drift and flexural yielding of columns.

2 MODEL DEVELOPMENT

The analytical model is based on stability functions which consider the reduction in stiffness from axial force due to second order effects. The model also incorporates inelastic column design curves which take into account the reduction in flexural stiffness due to initial residual stress effects. A brief description of the analytical model is summarized below. A detailed description of the theory behind stability functions and effective stiffness approaches are described in a research report (Peng 2006).

Stability functions were used to derive the global stiffness matrix for columns under an axial compression force, N, as shown in Figure 1. It relates the lateral deformations and rotations (υ_A, θ_A, υ_B, θ_B, υ_C, θ_C) to the shear forces and moments (V_A, M_A, V_B, M_B, V_C, M_C) at node A, B and C. As the lateral deformations at node A and C are zero, the overall expression can be simplified to a 4x4 matrix as shown in Equation 2 where s_1, g_1, r_1, f_1, s_2, g_2, r_2, f_2 are parameters that depend on the magnitude of the axial force. Note that in Figure 1, L_1 is much smaller than L_2 and M_A is greater than M_C.

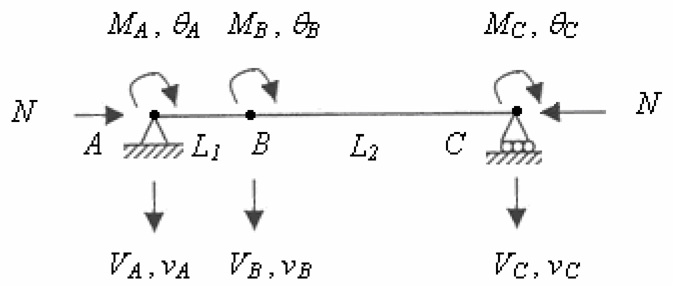


Figure 1. Column containing two sub-members and two internal degrees of freedom

$$EI\begin{bmatrix} s_1/L_1 & 0 & -g_1/L_1^{2} & r_1/L_1 \\ 0 & s_2/L_2 & g_2/L_2^{2} & r_2/L_2 \\ -g_1/L_1^{2} & g_2/L_2^{2} & f_1/L_1^{3}+f_2/L_2^{3} & -g_1/L_1^{2}+g_2/L_2^{2} \\ r_1/L_1 & r_2/L_2 & -g_1/L_1^{2}+g_2/L_2^{2} & s_1/L_1+s_2/L_2 \end{bmatrix}\begin{Bmatrix}\theta_A\\ \theta_C\\ v_B\\ \theta_B\end{Bmatrix}=\begin{Bmatrix}M_A\\ M_C\\ V_B\\ M_B\end{Bmatrix} \qquad (2)$$

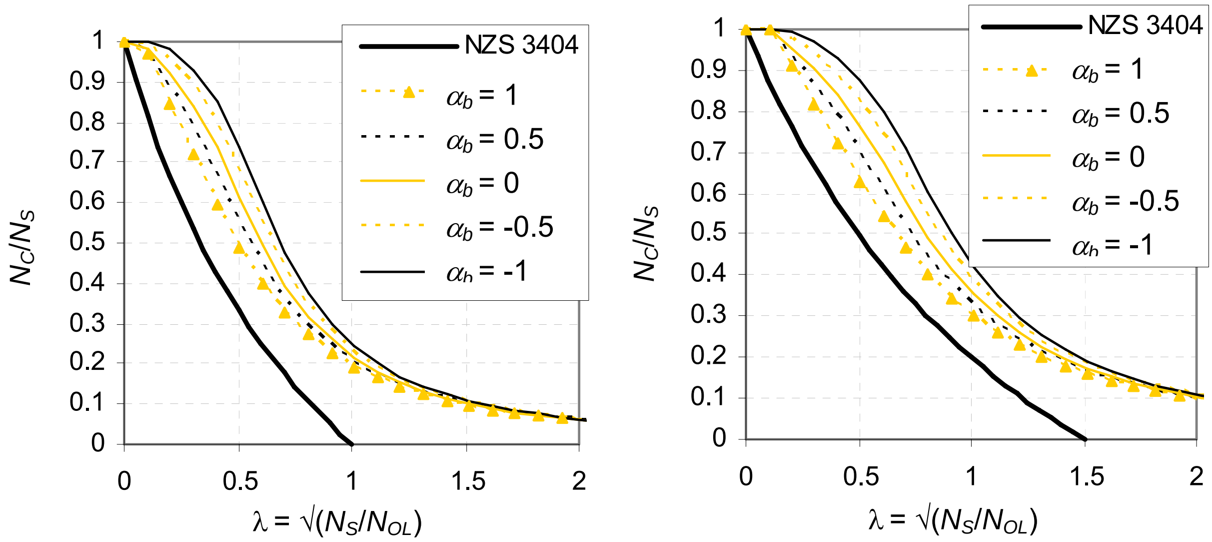
The reduction in flexural stiffness, $(EI)_t$, due to residual stress effects, is calculated by taking the ratio between the column design strength, N_C, and elastic buckling strength, N_{OL}, as shown in Equation 3 where E is Young's modulus and I is the second moment of area about the axis of bending. There are five column design curves in the New Zealand standard, representing different section types, described by section constants, α_b, which are taken as -1, -0.5, 0, 0.5, and 1. They were developed not only considering the residual stress effect, but also member out-of-straightness and accidental eccentric loading. Consequently, the effective flexural stiffness derived based on these curves may be more conservative than considering the residual stress effect alone.

$$(EI)_t = EI\left(\frac{N_C}{N_{OL}}\right) \tag{3}$$

The overall analytical procedure for determining the end yielding criteria, EYC, is iterative which requires the flexural stiffness to be updated as the axial force is changed. First, a simply supported member is chosen with specific properties such as end moment ratio, β_m, member slenderness, λ and section constant, α_b. A small axial force is applied and the effective flexural stiffness is calculated using Equation 3. The stability function, given in Equation 2, is then used in conjunction with the calculated effective flexural stiffness to determine the moment at node B, M_B. If M_B is less than M_A, the axial force is increased gradually until M_B is greater than M_A. The process is repeated for different slenderness limits, section constants, and end moment ratios so that a relationship between N_C/N_S, β_m, α_b, and λ can be obtained.

3 ANALYTICAL PREDICTIONS AND COMPARISONS

The analytical axial force ratio, that causes the maximum moment to move away from the member ends, is plotted together with the existing code provisions for different β_m as shown in Figure 2. It can be seen that five different curves are obtained from the analysis, representing different section types. As α_b increases, the initial stress effect in the member increases and consequently the ability of the member to carry axial force decreases. It can also be seen from these comparisons, that Lay's equation in the current seismic provision is more conservative than the results obtained in the analysis.



(a) $\beta_m = 0$ (b) $\beta_m = 0.5$

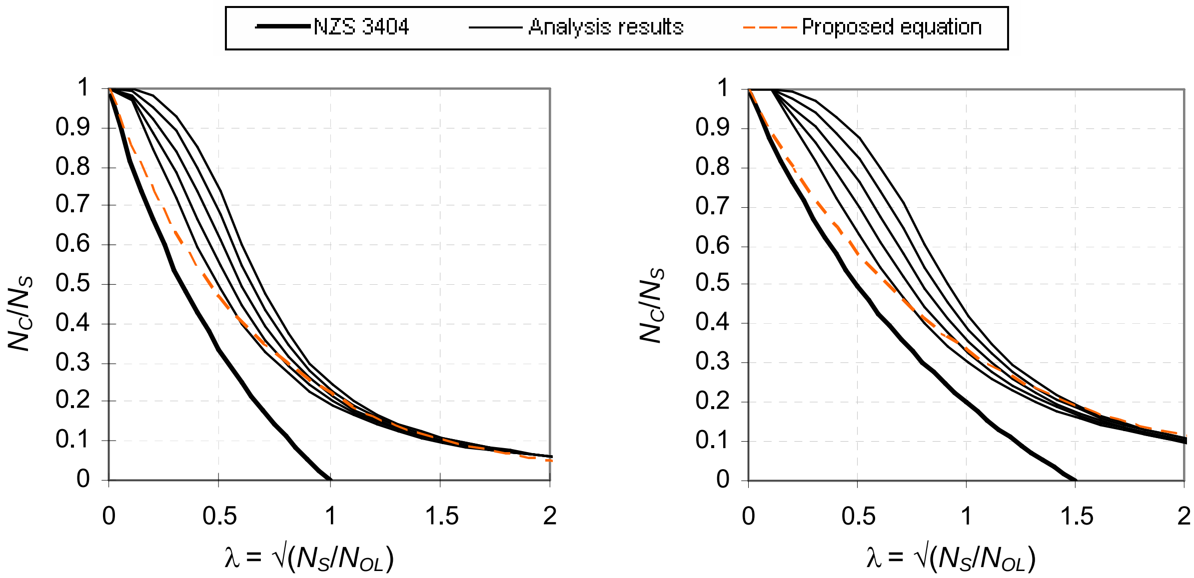
Figure 2. Comparison of EYC curves from the analysis and the New Zealand steel structures standard for different β_m

4 PROPOSED END YIELDING EQUATIONS

As there is no closed form solution for the analysis curves shown in Figure 2, an empirical equation given in Equation 4 is developed for the end yielding criteria. This equation is expressed as an exponential function which links the axial force ratio, N_C/N_S with the member slenderness limit, λ, and end moment ratio, β_m. Three constants, namely A, B and C are required for the EYC equation. These constants vary with different section types, α_b. It was suggested by Clifton (Peng 2006), that α_b should be taken as zero for design purposes as it is the most common section type used in design. For this case, A, B and C are equal to 0.263, 0.88 and 0.19 respectively. This simplification is slightly conservative for α_b less than zero and non-conservative for α_b greater than zero. The non-conservatism may be balanced out by the conservatism associated with the development of reduction in flexural stiffness.

$$\frac{N_C}{N_S} \leq \left\{ \frac{A \times (\beta_m + 1)^B}{e^{\left(C / \beta_m + 1 \right)}} \right\}^{\lambda} \tag{4}$$

The proposed EYC equation for α_b of zero is plotted together with the analysis results and the NZS3404 provision in Figure 3. It may be seen that the proposed EYC equation is more conservative than the analysis results, especially for columns with axial force ratios higher than 0.5. As the end moment ratio increases, the analysis curves become harder to fit using an exponential function. Subsequently, the proposed EYC equation becomes more conservative for members with higher axial loads. However, the proposed equation is still much closer to the analysis results than the current provisions in NZS3404. Adoption of the proposed EYC equation would relax the restriction on the column sizes specified in the current design standard.



(a) $\beta_m = 0$ (b) $\beta_m = 0.5$

Figure 3. Comparison of proposed equation, analysis results and NZS3404 provision for different β_m

5 APPLICATION OF EYC EQUATION

For yielding to occur away from the column ends, the column must i) violate the EYC equation and ii) yield in flexure. Columns which violate the EYC equation but which are not expected to yield should not be increased in size to prevent yielding away from the ends. A simple method is proposed to relate the axial force limit to inter-story drift and flexural yielding of columns which can be used in conjunction with the proposed EYC equation.

For the column to yield in flexure, the moment capacity must be smaller than the moment demand given by Equation 5 where $M_p(N)$ is the plastic moment capacity under axial force, M^* is the moment demand, F is the lateral force applied at the top of column, L is the height of column, K is the member stiffness which varies with end moment ratios, and Δ is the lateral displacement. Here, the column base is assumed to be fully fixed and the point of inflection is assumed to be at the top of the column.

$$M_P(N) \leq M^*(= FL = K\Delta L) \tag{5}$$

For an end moment ratio of zero, K is equal to $3EI/L^3$ and the equation can be simplified to Equation 6, where Δ/L is the inter-story drift.

$$M_P\left(1-\frac{N^*}{\phi N_s}\right) \leq \frac{3EI}{L^2}\Delta \tag{6}$$

This equation can be rearranged to obtain the axial force limit for flexural yielding, as a function of inter-story drift, as shown in Equation 7.

$$\frac{N^*}{\phi N_s} \geq 1-\frac{3EI}{M_P L}\left(\frac{\Delta}{L}\right) \tag{7}$$

Note that there are a few conservative assumptions associated with this derivation:

1) A conservative moment-axial force relationship is used.
2) Second order effects are not taken into consideration.
3) The effect of axial force on the stiffness is ignored.
4) It is assumed that all the lateral deformation arises from flexure. Lateral deformation arising from column lengthening/ shortening and shear deformation is neglected.

A schematic diagram of axial force limits for yielding and EYC is plotted in Figure 4. The region below the thick horizontal line corresponds to maximum moment occurring at the member ends and the region to the right of the thin diagonal line corresponds to flexural yielding. It may be seen that as the member moves from single curvature to double curvature, column yielding occurs at a lower drift and there is an increase in the axial force required to cause the maximum moment to move away from the member end.

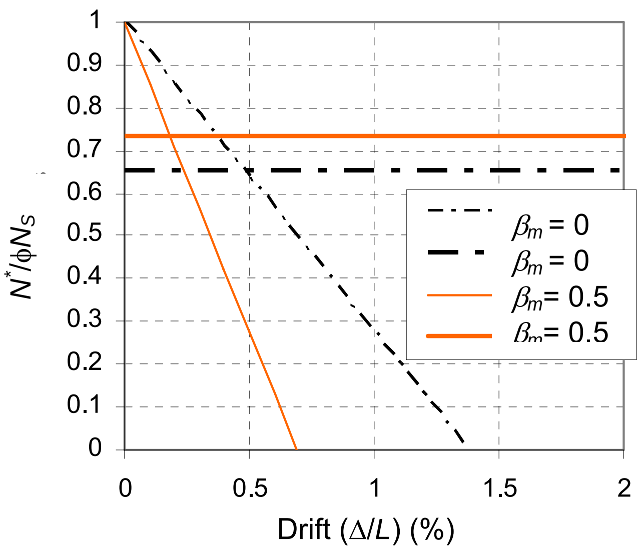


Figure 4. Yielding and EYC limit for different end moment ratios

6 TIME HISTORY ANALYSIS

Time history analysis was carried out to examine whether plastic hinges would occur away from the end of columns in EBFs, designed in accordance to the capacity design philosophy and the current seismic EYC equation. A 7 story EBF designed by the Heavy Engineering Research Association is used for the analysis. 14 earthquakes were obtained and scaled according to the New Zealand Loading Standard NZS1170.5:2004 (Standards New Zealand 2004).

It was found that the columns satisfy the capacity design method and the proposed EYC equation but failed under the current EYC equation. The distribution of the end moment ratio when the maximum moments or the maximum drifts occurred in the bottom story compression columns for 14 different earthquakes is plotted in Figure 5. It may be seen that the end moment ratios are between 0 and 0.4 when the maximum moments or drifts occur. This suggests that the $\beta_m = 0$ assumption used in the code is appropriate.

The axial force ratio and drift at the maximum moments and drifts are also plotted against the limiting line for yielding and EYC as shown in Figure 6. It may be seen that most columns remain elastic and the maximum moment generally occurred at the member ends. There is one point which lies just outside the elastic region. This contradicts the findings from RUAUMOKO where the columns were found to remain elastic for all 14 earthquakes. This is due to the conservative assumption used to derive the proposed yield line.

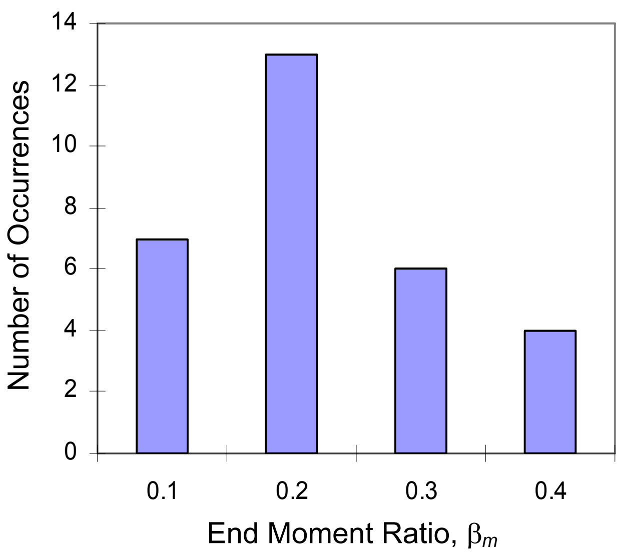


Figure 5. End moment ratio distribution in bottom story columns at maximum moment or maximum drift

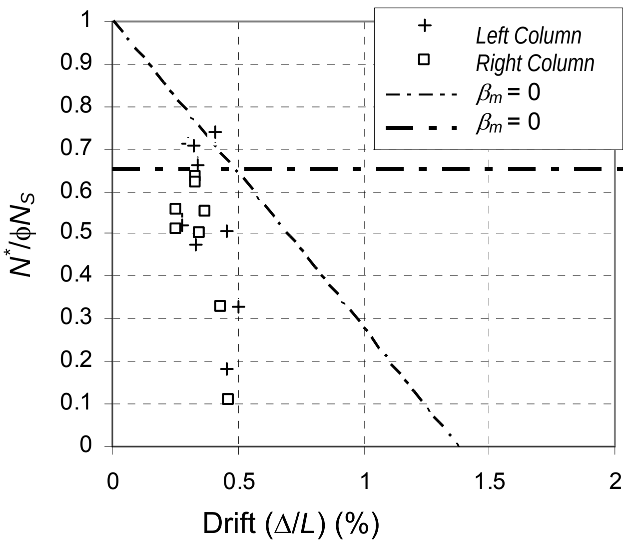
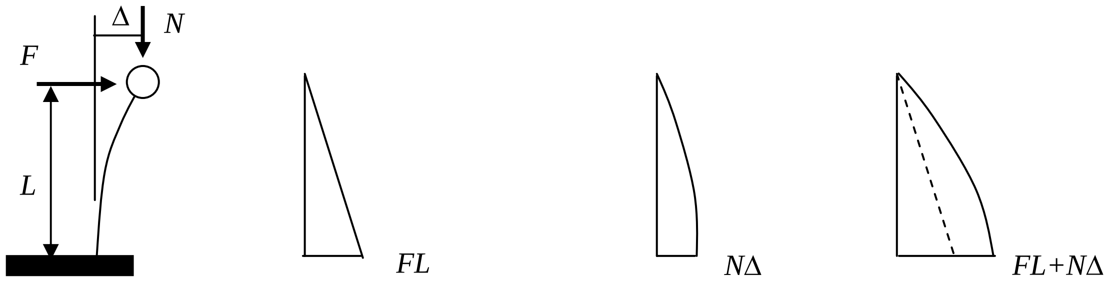


Figure 6. Axial force ratios and drifts at the maximum moments or drift

7 APPLICABILITY OF EYC EQUATION ON STEEL FRAMES

The EYC equation developed in this study and the one proposed by Lay were based on the assumption that the ends of columns are fixed in place and the forces are applied axially along the member as shown in Figure 1. In this case the second-order moments will follow the deformed shape and consequently the maximum moment may occur away from the column ends.

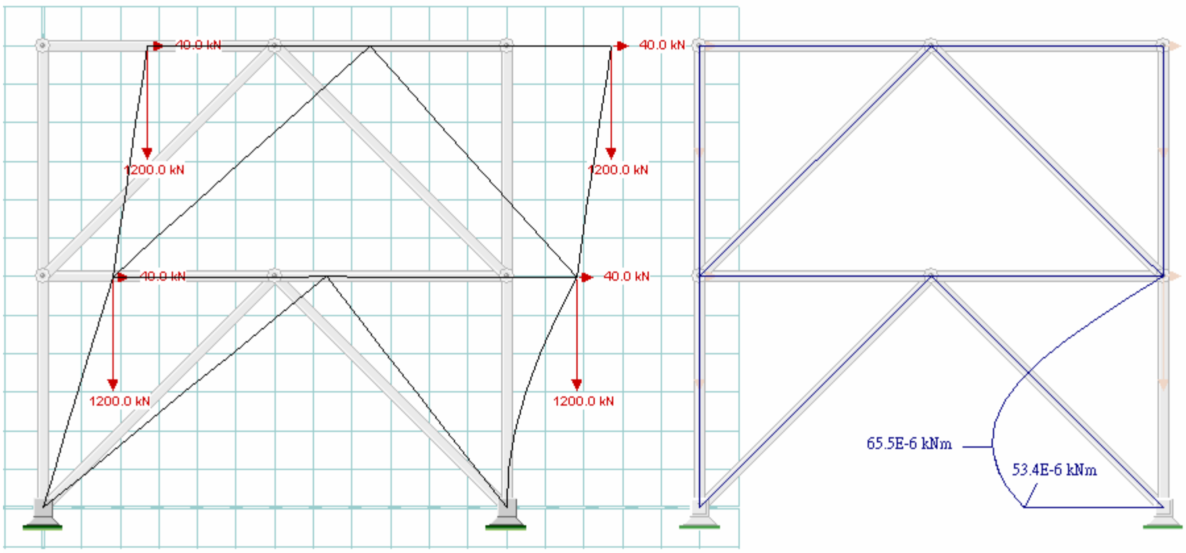
For moment resisting frames, the load resisting mechanisms and the end restraints are quite different from the ones used for developing the EYC equation. The bottom story columns in a moment resisting frame act in a similar manner to a cantilever as shown in Figure 7. It can be seen that both the first order and second order moments are a maximum at the base. Therefore, the maximum moment can not occur away from the member ends and the EYC equation should not be applied to columns of these frames.



(a) Column (b) First order moments (c) Second order moments (d) Total moments

Figure 7. Bottom story column forces and moments in moment resisting frames

For eccentrically and concentrically braced frames, the end restraints are similar to the ones developed for the EYC. A two story pin-connected CBF model was set up in Dr. Frame to examine the bending moment profiles in the bottom story columns as shown in Figure 8a. All the members apart from the bottom right hand column are truss elements that can only carry axial force but not moment. The right hand bottom story column was provided with a small flexural stiffness and was fixed at the base. It can be seen from Figure 8b that the maximum moment occurred away from the member ends. Therefore, the EYC equation should be applied to columns in eccentrically and concentrically braced frames.



(a) Applied force and deformed shape (b) Internal forces and moments

Figure 8. Braced frame analysis with moment fixity at the base of the right hand column (Dr. Software 1998)

8 CONCLUSIONS

Based on the outcomes of this study, the following conclusions can be drawn:

1) Current seismic code provisions in the New Zealand steel structures standard for end yielding criteria are found to be conservative.
2) A new design equation is proposed in this study to represent the axial force ratio causing the plastic hinges to move away from the member ends. The proposed equation is a significantly better match to the end yielding criteria curves obtained from the analysis. This equation forms the basis for proposed amendments to Clause 12.8.3.1 in NZS3404.
3) The assumption of using $\beta_m = 0$ for design with the seismic provisions is appropriate.
4) A simple method is proposed to determine whether columns will yield for a given design drift and axial force ratio. This should be used in conjunction with the proposed EYC equation for the design of columns in steel frames.
5) The EYC equation is not applicable to columns in moment resisting frames. On the other hand, it should be applied to columns in eccentrically and concentrically braced frames.

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